PMM U.S.S.R.,Vol.45,pp.561-563 0021-8928/82/4 0561 $7.50/0

UDC 629.113+629.735.33

SELF-OSCILLATIONS OF A WHEEL ON SELF-ORIENTATING STRUT OF AN UNDERCARRIAGE WITH NONLINEAR DAMPER*

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Under the assumptions of the drift hypothesis the perturbation methods are used to determine the amplitude and frequency of free oscillations of a wheel on a self-orientating strut subjected to flexural deformations. The strut is fitted with a square damper generating a turbulent resistance and coupled in parallel with an elastic element. When such a system reaches a certain velocity, it becomes potentially self-oscillating. A limiting cycle is found and an example of computing the amplitude and frequency of the self-oscillations is given.

The oscillations of the system are described by two, uniformly moving oscillators, the rotation of the wheel providing a gyroscopic coupling between them as well as a directed coupling determined by the reaction of the runway on the foot of the undercarriage

$$\begin{aligned} &B\theta^{\cdot\cdot} + h\theta^{\cdot} + k_1\theta - Ivr^{-1}\psi^{\cdot} = -c_d\theta^{\cdot 2}\operatorname{sign}\theta^{\cdot} \\ &C\psi^{\cdot\cdot} + h_1\psi^{\cdot} + k\psi + Ivr^{-1}\theta^{\cdot} = -a_1 l\theta \end{aligned} \tag{1}$$

Here θ and ψ are the angles of yaw and roll, B and C are the moments of inertia of the undercarriage relative to the axis of strut and roll respectively, h and h_1 are the linear viscous friction coefficients, k_1 and k are rigidities, I and r denote the axial moment of inertia and the wheel radius respectively, v is velocity of motion, a_1 is the drift resistance coefficient, c_d is the damper resistance coefficient and l is the distance between the runway surface and the axis of roll.

The gyroscopic and directed coupling form, in the system in question, a cycle which may lead to instability of the rectilinear motion and self-induced oscillation of the foot of the undercarriage (shimmy). The engine represents the external energy source responsible for the directed coupling. The eigenvalues of the matrix of the linear part of the system (1) have negative real parts when

$$l < l_* = (h/B + h_1/C)^{1/2}(hk + h_1k_1)^{1/2}/a_1$$

When $l > l_*$, a value v_0 of the velocity of motion exists such that when $v < v_0$, then the linear system remains asymptotically stable. The real parts of two eigenvalues are positive of $v \in (v_0, v_0')$. Moreover, the value of v_0 decreases and that of v_0' increases with increasing l. Thus the asymptotic stability of a linear system is replaced, during the passage through v_0, by instability. Since the condition $l < l_*$ cannot be realized in practice, an additional square damper must be used /2/, and to determine its resistance coefficient and the magnitude of admissible clearance, the amplitudes and frequencies of the resulting self-oscillations must also be found. Below we solve the latter problem in the first approximation, using the asymptotic method of expanding the derivatives /3/. Putting $x_1 = \theta,\ x_2 = \theta^{\cdot},\ x_3 = \psi,\ x_4 = \psi^{\cdot}$, we write the system (I) in the form

$$\mathbf{x}^{\cdot} = A\mathbf{x} - \mathbf{d}x_2^2\operatorname{sign} x_2 \tag{2}$$

$$A = \begin{Vmatrix} 0 & 1 & 0 & 0 \\ -E_\theta & -\varepsilon_\theta & 0 & Dv \\ 0 & 0 & 0 & 1 \\ -H_1 & -D_1v & -E_\psi & -\varepsilon_\psi \end{Vmatrix}, \quad \mathbf{x} = \begin{Vmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Vmatrix}, \quad \mathbf{d} = \begin{Vmatrix} 0 \\ k_d \\ 0 \\ 0 \end{Vmatrix}$$

$$\varepsilon_\theta = h/B,\ E_\theta = k_1/B,\ D = IB^{-1}r^{-1},\ k_d = C_d/B$$
$$\varepsilon_\psi = h_1/C,\ E_\psi = k/C,\ D_1 = IC^{-1}r^{-1},\ H_1 = a_1 l/C$$

Let us set $v = v_0 + \varepsilon v_1 + O(\varepsilon^2)$. To find the amplitude and frequency of the self-oscillations, we shall seek a solution to the equation (2) for $v > v_0$, in the form

$$\mathbf{x} = \varepsilon\mathbf{q}_0 + \varepsilon^2\mathbf{q}_1 + \ldots,\quad \mathbf{q}_0 = \operatorname{col}(q_{10}, \ldots, q_{40}),\quad \mathbf{q}_1 = \operatorname{col}(q_{11}, \ldots, q_{41}), \ldots$$

*Prikl.Matem.Mekhan.,45,No.4,756-759,1981

Here $\varepsilon = (v - v_0)/v_1 + O(\varepsilon^2)$, $\varepsilon > 0$ when $v > v_0$ and $\varepsilon = 0$ when $v = v_0$. Let us set $t_0 = t$ and introduce the slow time variables $t_i = \varepsilon^i t$, $i = 1, 2, \ldots$. Then $d/dt = \partial/\partial t_0 + \varepsilon\partial/\partial t_1 + \varepsilon^2\partial/\partial t_2 + \ldots$ and we have the following expressions for determining $\mathbf{q}_1$:

$$L\mathbf{q}_0 = 0, \quad L\mathbf{q}_1 = -\partial\mathbf{q}_0/\partial t_1 + v_1 A_1 \mathbf{q}_0 - \mathbf{d} q_{20}^2 \operatorname{sign} q_{20}, \ldots \qquad (3)$$
$$L \equiv \partial/\partial t_0 - A_0, \quad A_0 = A|_{v=v_0}$$

Here A_1 is a matrix the only nonzero element of which is $(A_1)_{42} = -D_1$. The eigenvalues of the matrix A_0 are $p_{1,2} = \pm i\Omega_0$, $\operatorname{Re} p_{3,4} < 0$, and the right eigenvector corresponding to the eigenvalue $i\Omega_0$ is collinear with the vector

$$\mathbf{u} = \operatorname{col}\{1, i\Omega_0, [\varepsilon_\theta\Omega_0 + i(\Omega_0^2 - E_\psi)] D^{-1}v_0^{-1}\Omega_0^{-1}, (E_\theta - \Omega_0^2 + i\varepsilon_\theta\Omega_0) D^{-1}v_0^{-1}\}$$
$$\Omega_0^2 = (\varepsilon_\theta E_\psi + \varepsilon_\psi E_\theta + DH_1 v_0)(\varepsilon_\theta + \varepsilon_\psi)^{-1}$$

where Ω_0 is the frequency of the linear system. From the first equation of (3) we have

$$\mathbf{q}_0 = 2a\operatorname{Re}[\mathbf{u}\exp(i\varphi)]$$
$$\varphi = \Omega_0 t_0 + \varphi_0, \quad a = a(t_1, t_2, \ldots), \quad \varphi_0 = \varphi_0(t_1, t_2, \ldots)$$

Let us write the last term of the second equation of (3) in terms of a complex Fourier series

$$\mathbf{d} q_{20}^2 \operatorname{sign} q_{20} = \sum_{m=-\infty}^{+\infty} a^2 \mathbf{c}_m \exp(im\varphi) \qquad (4)$$
$$a^2\mathbf{c}_m = \frac{\mathbf{d}}{2\pi}\int_0^{2\pi} q_{20}^2 \operatorname{sign} q_{20} \exp(-im\varphi)\, d\varphi$$

From (3) it follows that $\mathbf{q}_1$ is given by the equation

$$\partial\mathbf{q}_1/\partial t_0 - A_0\mathbf{q}_1 = \Phi(\mathbf{u}, i, \mathbf{c}_1)\exp[i(\Omega_0 t_0 + \varphi_0)] + \Phi(\bar{\mathbf{u}}, -i, \mathbf{c}_{-1})\exp[-i(\Omega_0 t_0 + \varphi_0)] - a^2 \sum_{\substack{m=-\infty \\ m \neq \pm 1}}^{+\infty} \mathbf{c}_m \exp(im\varphi)$$
$$\Phi(y, i, y_1) = y(-\partial a/\partial t_1 - ia\partial\varphi/\partial t_1) + v_1 a A_1 y - a^2 y_1$$

Let $\mathbf{U} = (U_1, \ldots, U_4)$ be the first eigenvector of the matrix A_0 corresponding to the eigenvalue $i\Omega_0$ i.e.

$$\mathbf{U}(A_0 - i\Omega_0 E) = 0 \qquad (5)$$

We introduce the scalar $z = \mathbf{U}\mathbf{q}_1$, and hence obtain $\mathbf{U}A_0\mathbf{q}_1 = i\Omega_0 z$. The expression for z, and hence for $\mathbf{q}_1$, will not contain secular terms if

$$-\partial a/\partial t_1 - ia\partial\varphi/\partial t_1 + a(\beta + i\beta_1) - a^2(\gamma + i\gamma_1) = 0 \qquad (6)$$
$$\beta + i\beta_1 = v_1\alpha^{-1}\mathbf{U}A_1\mathbf{u}, \quad \gamma + i\gamma_1 = \alpha^{-1}\mathbf{U}\mathbf{c}_1, \quad \alpha = \mathbf{U}\mathbf{u}$$

Therefore

$$a = \beta[K\exp(-\beta t_1) + \gamma]^{-1}, \quad \varphi = \Omega_0 t_0 + (\beta\gamma_1/\gamma - \beta_1)t_1 + \delta \qquad (7)$$
$$\delta = \gamma_1\gamma^{-1}\ln[K\exp(-\beta t_1) + \gamma] + \varphi_*(t_2, t_3, \ldots), \quad K = K(t_2, t_3, \ldots)$$

and hence

$$\theta = 2\varepsilon a\cos\varphi + O(\varepsilon^2), \quad \theta' = -2\varepsilon a\Omega_0\sin\varphi + O(\varepsilon^2)$$
$$\psi = 2\varepsilon D^{-1}v_0^{-1}a[\varepsilon_\theta\cos\varphi + (E_\theta\Omega_0^{-1} - \Omega_0)\sin\varphi] + O(\varepsilon^2)$$
$$\psi' = 2\varepsilon D^{-1}v_0^{-1}a[(E_\theta - \Omega_0^2)\cos\varphi - \varepsilon_\theta\Omega_0\sin\varphi] + O(\varepsilon^2)$$

The periodic solution is established in the system after a certain time has elapsed. To obtain such a solution, we must assume the time to be sufficiently long, and to separate, from amongst the limiting cycles, the stable cycles. In accordance with the procedure of the asymptotic multiscale method, the period of time after which the self-oscillatory region is established, corresponds to $t_1 \to +\infty$, i.e. the conditions of existence of a limiting cycle are determined by the behavior of the function $a(t_1)$ at infinity.

For a linear system $\mathbf{d} = 0$, therefore $\mathbf{c}_m = 0$, $\gamma = 0$, $\gamma_1 = 0$. This implies that $a = \beta K\exp(\beta t_1)$. When $v < v_0$, the linear system is asymptotically stable, consequently $\beta < 0$ when $v < v_0$. Since the linear system is unstable when $v > v_0$, we have $\beta > 0$ when $v > v_0$.
For the nonlinear system of (7) we obtain, for the case $\gamma > 0$, $\beta < 0$, $\lim_{t_1\to+\infty} a = 0$, $\beta > 0$, $\lim_{t_1\to+\infty} a = \beta/\gamma$. Let us consider the nonlinear system for the case $\gamma < 0$. The behavior of the function $a(t_1)$ at $\beta < 0$ is shown in Fig's.1 and 2. If the initial value of the amplitude is smaller than the amplitude of the limiting cycle, then the motion is stable since $\lim_{t_1\to+\infty} a = 0$ when $a|_{t_1=0} = \beta(K+\gamma)^{-1} < \beta/\gamma$ (Fig.1). Otherwise the motion is unstable since $\lim_{t_1\to t_{1*}} a = +\infty$ when $a|_{t_1=0} > \beta/\gamma$

(Fig.2). (Here $t_{10} = -\beta^{-1}\ln(-\gamma/K)$). The results depicted in Fig.3 imply that the motion is unstable when $\beta > 0$. Thus if $\gamma < 0$, then we have an unstable limiting cycle when $\beta < 0$ $(v < v_0)$ while the motion is unstable when $\beta > 0$.

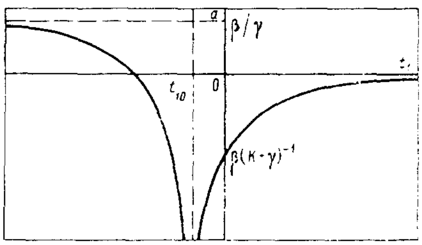

Fig.1

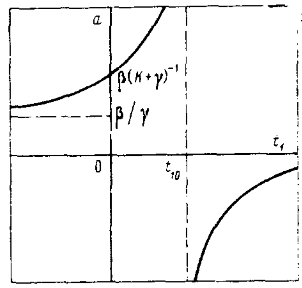

Fig.2

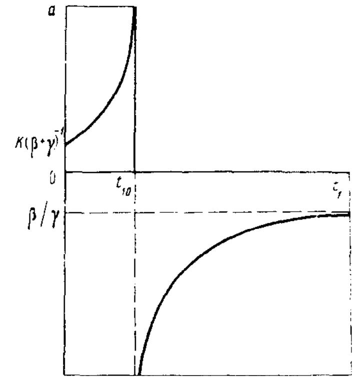

Fig.3

The limiting cycle in question forms the boundaries of the region of attraction of the null solution. In the case of $\gamma > 0$ the motion is stable when $\beta < 0$, and a stable limiting cycle exists when $\beta > 0$. We have for this cycle

$$\theta = \Theta\cos(\Omega t_0 + \delta_0) + O(\varepsilon^2),\ \Theta = 2(v - v_0)v_1^{-1}\beta\gamma^{-1}$$
$$\Omega = \Omega_0 - (v - v_0)v_1^{-1}(\beta\gamma_1/\gamma - \beta_1),\ \delta_0 = \text{const}$$

From (4)—(6) we obtain

$$\mathbf{c}_1 = 16i(3\pi)^{-1}\Omega_0^2\mathbf{d},\ \mathbf{U} = \{U_1, U_2, U_3, U_4\}$$
$$U_1 = \varepsilon_\theta\varepsilon_\psi + DD_1v_0^2 + E_\psi - \Omega_0^2 + i(\varepsilon_\theta\Omega_0 - \varepsilon_\theta E_\psi/\Omega_0 + \varepsilon_\psi\Omega_0)]D^{-1}v_0^{-1}$$
$$U_2 = [\varepsilon_\psi + i(\Omega_0 - E_\psi/\Omega_0)]D^{-1}v_0^{-1},\ U_3 = iE_\psi/\Omega_0,\ U_4 = 1$$
$$\alpha = (\alpha_{01} + i\alpha_{11})/(Dv_0\Omega_0^2)$$
$$\alpha_{01} = -3\Omega_0^4 + (E_\theta + E_\psi + \varepsilon_\theta\varepsilon_\psi + DD_1v_0^2)\Omega_0^2 + E_\theta E_\psi \quad \alpha_{11} = 2\Omega_0^3(\varepsilon_\theta + \varepsilon_\psi)$$
$$\gamma = 16(3\pi)^{-1}k_d\Omega_0^3\sigma[\alpha_{01}(E_\psi - \Omega_0^2) + \alpha_{11}\varepsilon_\psi\Omega_0]$$
$$\gamma_1 = 16(3\pi)^{-1}k_d\Omega_0^3\sigma[\alpha_{01}\varepsilon_\psi\Omega_0 + \alpha_{11}(\Omega_0^2 - E_\psi)] \quad \beta = v_1\Omega_0^2v_0^{-1}\sigma(\alpha_{01}b_1 + \alpha_{11}b_2)$$
$$\beta_1 = v_1\Omega_0^2v_0^{-1}\sigma(\alpha_{01}b_2 + \alpha_{11}b_1),\quad \sigma = (\alpha_{01}^2 + \alpha_{11}^2)^{-1}$$
$$b_1 = \varepsilon_\psi(E_\theta - \Omega_0^2) + \varepsilon_\theta(E_\psi - \Omega_0^2)$$
$$b_2 = \Omega_0(\varepsilon_\theta\varepsilon_\psi - DD_1v_0^2) - \Omega_0^{-1}(\Omega_0^2 - E_\theta)(\Omega_0^2 - E_\psi)$$

For the particular numerical values of $B = 9.81$ kg m^2, $C = 165$ kg m^2, $h = 37$ m.s , $h_1 = 981$ m.s , $k_1 = 12200$ m, $k = 421000$ m, $I = 11.8$ kg m^2, $r = 0.4$ m, $l = 0.8$ m, $a_1 = 42670$ n, we have $v_0 = 17.96$ m.s^{-1} and $\Omega_0 = 53.8$ s^{-1}. The dependence of the shimmy amplitude Θ on $v - v_0$ is linear within the given approximation. In the present case the proportionality coefficient for the values of $k_d = 1.0$, 3.0, 5.0 and 10.0 equals, respectively, to $2\cdot10^{-2}$, $4.97\cdot10^{-3}$, $3.23\cdot10^{-3}$, $1.29\cdot10^{-3}$ rad.s/m. The shimmy amplitude decreases with increasing value of k_d of the square damper. The shimmy period $T = 2\pi/\Omega$ is independent of the magnitude of k_d; for the present case we have $T = \omega_1(v - v_0) + \omega_0$, $\omega_0 = 8.12\cdot10^{-2}$ s^{-1} and $\omega_1 = 5.17\cdot10^{-4}$ cm^{-1}. The frequency Ω of the nonlinear system is smaller than the frequency Ω_0 of the linear system. The shimmy amplitude can be controlled so as to keep it within the safe limits, by choosing the corresponding coefficient of resistance of the square damper.

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Translated by L.K.